LETS GET READY – HIGHER TIER - SOLUTIONS

1st May

Find the length of side AB

AB = 9 ÷ cos60

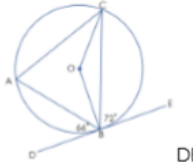
cos 60 = ½
AB = 9 ÷ ½ = **18cm**

2nd May

Show that
$(3x-2)(2x+3)(x-5)$
can be written in the form
ax^3+bx^2+cx+d
Where a, b, c and d are integers to be found.

$(3x – 2)(2x + 3) = 6x^2 + 9x – 4x – 6 = 6x^2 + 5x – 6$
$(6x^2 + 5x – 6)(x – 5) = 6x^3 – 30x^2 + 5x^2 – 25x – 6x + 30$
$= 6x^3 – 25x^2 – 31x + 30$
a = 6, b = -25, c = -31, d = 30

3rd May



DE is a tangent to the circle with centre O. Angle ABD = 66° and angle CBE = 72°. Find angle ACO giving full geometric reasons

OBE = 90° (radius and tangent meet at a right angle)
OBC = 18°
OCB = 18° (base angles of an isosceles triangle are equal)
ACB = 66° (alternate segment theorem)
AOC= 66° - 18° = **48°**

4th May

Make m the subject of the formula

$$r=\frac{4m-5}{9-3m}$$

r(9 – 3m) = 4m – 5
9r – 3mr = 4m – 5
9r + 5 = 4m + 3mr 9r + 5 = m(4 + 3r)
$\boldsymbol{m=\frac{9r+5}{4+3r}}$

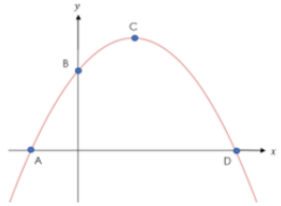
5th May

Find the answer to

$$\left(5\frac{3}{5}-2\frac{1}{6}\right)\div 1\frac{2}{5}$$

$\left(\frac{28}{5}-\frac{13}{6}\right)\div\frac{7}{5}$ = $\left(\frac{168}{30}-\frac{65}{30}\right)\times\frac{5}{7}$ $\frac{103}{30}\times\frac{5}{7}$ = $\boldsymbol{\frac{103}{42}}$

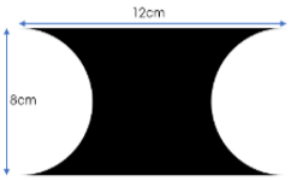
6th May



This is the graph of $y = 10 + 3x – x^2$. Find the coordinates A, B, C and D

A and D – y=0 $10 + 3x – x^2 = 0$, $x^2 – 3x – 10 = 0$
(x + 2)(x – 5) = 0, x = -2 and 5, **A(-2, 0) D(5, 0)**
B – x=0, y = 10 **C(0, 10)**
Complete the square: $y = -(x^2 – 3x – 10) = -[(x – {}^3/_2) – {}^9/_4 – 10]$
$y = -[(x – {}^3/_2) – {}^{49}/_4] = -(x – {}^3/_2) + {}^{49}/_4$ **C($^3/_2$, $^{49}/_4$)**

7th May



The diagram shows a rectangle with two semi-circles removed from the sides.
Giving your answers in terms of π,
(a) Find the perimeter of the shape
(b) Find the area of the shape

(a) Curved lengths = circumference of 1 circle = 8π
Perimeter = 12 + 12 + 8π = **24 + 8π cm**

(b) Area of rectangle = 12 x 8 = 96cm^2
2 semi-circles = 1 circle Area = π x 4^2 = 16π
Shaded area = **96 –16π cm^2**

8th May

Write the decimal $2.1\dot{4}0\dot{5}$ as an improper fraction in its simplest form.

$X = 2.1\dot{4}0\dot{5}$ $10X = 21.\dot{4}0\dot{5}$ (1) $10000X = 21405.\dot{4}0\dot{5}$ (2)
(2) – (1) $9990X = 21384$ $X = \frac{21384}{9990}$ $\boldsymbol{X = \frac{396}{185}}$

9th May

(a) f(x) = 4x – 5. Find f(5)
(b) g(x) = 3x^2 – 2. Find fg(-4)
(c) Solve the equation
f^{-1}(x) = 7x + 3

(a) f(5) = 4(5) – 5 = **15**
(b) g(-4) = 3(-4)^2 – 2 = 3(16) – 2 = 46 fg(-4) = f(46) = 4(46) – 5 = **179**
(c) $f^{-1}(x) = \frac{x+5}{4}$ $\frac{x+5}{4} = 7x + 3$ $x + 5 = 28x + 12$
$5 = 27x + 12$ $-7 = 27x$ $\boldsymbol{x = -\frac{7}{27}}$

10th May

The ratio of a : b is 5 : 9
The ratio of b : c is 4 : 11.
Write the ratio of a : b : c where a, b and c are integers and a : b : c is a ratio in its simplest form.

a : b = 5 : 9 = $^5/_9$: 1
b : c = 4 : 11 = 1 : $^{11}/_4$

a : b : c = $^5/_9$: 1 : $^{11}/_4$ Common denominator = 9 x 4 = 36
x 36: **a : b : c = 20 : 36 : 99**

11th May

On the graph of y = f(x) point A has coordinate (4, -3).
What would the coordinate of A be under the following transformations
(a) f(x + 3)
(b) f(x) + 8
(c) f(2x)
(d) 3 – 4f(x)

(a) - 3 from x: **(1, -3)**
(b) +8 to y: **(4, 5)**
(c) ÷ x by 2 **(2, -3)**
(d) Multiply y by 4. 3 subtract this for new y-coordinate **(4, 15)**

12th May

A is indirectly proportional to the square of B.
When A = 48, B = $\frac{1}{4}$.
Find B when A = 75

$A \propto \frac{1}{B^2}$ $A = \frac{k}{B^2}$ A=48, k=$^1/_4$ $48 = \frac{k}{^1/_{16}}$ k = 3
$A = \frac{3}{B^2}$ $75 = \frac{3}{B^2}$ $B^2 = \frac{3}{75} = \frac{1}{25}$ $\boldsymbol{B = \frac{1}{5}}$

13th May

If $\begin{pmatrix} 5a+2b \\ 2a-3b \end{pmatrix} = 4\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ find the values of a and b

5a + 2b = 12 (1) 2a – 3b = 20 (2)
(1)x2: 10a + 4b = 24 (3) (2)x5: 10a – 15b = 100 (4)
(3) – (4): 19b = -76 **b = -4**
Sub in (1): 5a – 8 = 12 5a = 20 **a = 4**

14th May

(a) Solve the inequality
$7x + 6 < 48$
(b) Solve the inequality
$3x^2 - 16x - 35 \leq 0$
(c) Write down the integer values which satisfy the solution to (a) **and** (b)

(a) 7x < 42 **x < 6**
(b) (3x + 5)(x – 7) ≤ 0 Roots: x = $-5/3$ and 7 $-5/3 \leq x \leq 7$
(c) -1, 0, 1, 2, 3, 4, 5

15th May

A model of a statue is made which is mathematically similar to the statue. The volume of material needed to make the statue is 125m³. The model requires 27m³ of material. The model needs 12m² of paint to cover it. How much will the statue need?

Volume SF = $\frac{125}{27}$ Length SF = $\sqrt[3]{\frac{125}{27}} = \frac{5}{3}$
Area SF = $\left(\frac{5}{3}\right)^2 = \frac{25}{9}$ Paint needed = $\frac{25}{9} \times 12 = \mathbf{\frac{100}{3}}$ **cm²**

16th May

(a) Write 260 as a product of prime factors
Hence, or otherwise
(b) Find the highest common factor of 260 and 200
(c) Find the lowest common multiple of 260 and 300

(a) 260 = **2 x 2 x 5 x 13 or 2^2 x 5 x 13**
(b) 200 = 2 x 2 x 2 x 5 x 5
260 = 2 x 2 x 5 x 13
HCF = 2 x 2 x 5 = **20**
(c) 200 = 2 x 2 x 2 x 5 x 5
260 = 2 x 2 x 5 x 13
LCM = 2 x 2 x 2 x 5 x 5 x 13 = **2600**

17th May

The diagram shows a circle inside a square – the sides of the square are tangents to the circle. The square has an area of 144cm². Work out the shaded area in terms of π.

length of side of square = diameter of circle = 12cm
Area of circle = π x 6^2 = 36π
Shaded area = **144 - 36π**

18th May

(i) Write down the exact value of
(a) 3^{-3}
(b) $\left(\frac{25}{16}\right)^{-3/2}$
(ii) Solve the equation
$9^{x+1} \times 27^{2x-1} = 3^{5x+4}$

(i) (a) $\frac{1}{27}$ (b) $\left(\frac{16}{25}\right)^{3/2} = \left(\frac{4}{5}\right)^3 = \mathbf{\frac{64}{125}}$
(ii) $(3^2)^{x+1} \times (3^3)^{2x-1} = 3^{5x-4}$
$3^{2x+2} \times 3^{6x-3} = 3^{5x-4}$ $3^{(2x+2)+(6x-3)} = 3^{5x-4}$
$3^{8x-1} = 3^{5x-4}$ 8x – 1 = 5x – 4 3x = -3 **x = -1**

19th May

The curve C with equation
$y = 3x^2 - 15x - 15$
Intersects with the line L with equation
$y + 4x = 5$
At points A and B.
Find the coordinates of A and B.

y = 5 – 4x $5 - 4x = 3x^2 - 15x - 15$ $3x^2 - 11x - 20 = 0$
(3x + 4)(x – 5) = 0 x = $-4/3$ and 5
y = 5 – 4($-4/3$) = $31/3$ **A($-4/3$, $31/3$)**
y = 5 – 4(5) = -15 **B(5, -15)**

20th May

The population of Hillsborough is declining at a rate of 10% each year. Currently the population of Hillsborough is 27,000.
(a) What was the population last year?
(b) What will the population be next year if this decline continues?

(a) Last year -> x 0.9 -> Next Year
Next year -> ÷ 0.9 -> Last year
27000 -> ÷ 0.9 -> **30,000**

(b) 30,000 ÷ 0.9 = 300000 ÷ 9 = 33,333

21st May

Solve the pair of simultaneous equations
$9x + 5y = 26$
$5x - 7y = 34$

9x + 5y = 26 (1) 5x – 7y = 34 (2)
(1)x5: 45x + 25y = 130 (3) (2)x9: 45x – 63y = 306 (4)
(3) – (4): 88y = -176 **y = -2**
Sub in (1): 9x + 5(-2) = 26 9x – 10 = 26 **x = 4**

22nd May

A cuboid has its length increased by 20%, its width decreased by 30% and its height increased by 10%.
What is the overall effect on the volume?

Length: 1.2 Width: 0.7 Height: 1.1
Change = 1.2 x 0.7 x 1.1 = 0.84 x 1.1 = 0.924 = 92.4%
Decrease of 7.6%

23rd May

(a) Write $6\sqrt{54} - 5\sqrt{24}$ in the form $a\sqrt{b}$ where a and b are integers.
(b) Expand and simplify fully
$(4 - 3\sqrt{7})^2$
(c) Rationalise and simplify fully
$\frac{3 + 5\sqrt{2}}{4\sqrt{6}}$

(a) $6\sqrt{9 \times 6} - 5\sqrt{4 \times 6} = 18\sqrt{6} - 10\sqrt{6} = \mathbf{8\sqrt{6}}$
(b) $(4 - 3\sqrt{7})(4 - 3\sqrt{7}) = 16 - 12\sqrt{7} - 12\sqrt{7} + 9(7) = \mathbf{79 - 24\sqrt{7}}$
(c) $\frac{(3+5\sqrt{2})\sqrt{6}}{4\sqrt{6}\times\sqrt{6}} = \frac{3\sqrt{6}+5\sqrt{12}}{4\times6} = \frac{\mathbf{3\sqrt{6}+10\sqrt{3}}}{\mathbf{24}}$

25th May

The diagram shows a child's toy made of a hemisphere and a cone. The density of the material used to make the toy is 1.1g/cm³.
What is the mass of the toy?

Cone Volume = $\frac{1}{3} \times \pi \times 3.5^2 \times 5.5 = 70.555$ cm³
Hemisphere Volume = $\frac{1}{2} \times \frac{4}{3} \times \pi \times 3.5^3 = 89.797$ cm³
Total Volume = 160.352cm³
Mass = Density x Volume = 1.1 x 160.352 = 176.4g

26th May

A graph is drawn of the function
$y = a \times b^x$
Where a and b are constants. The graph passes through the points with coordinates (3, 64) and (6, 512).
Find the values of a and b.

x = 3, y = 64: 64 = a x b³ (1)
x = 6, y = 512: 512 = a x b⁶ (2)
(2) ÷ (1): 8 = b³ **b = 2**
Sub in (1) 64 = a x 2³ **a = 8**

27th May

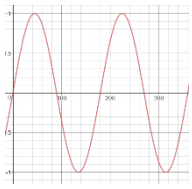
Joan invested £5000 in a bank account three years ago. The bank pays x% compound interest per year.
She currently has £5431.87 in her account. She has not withdrawn or deposited any more money. Find the value of x to 1 decimal place.

5431.87 = 5000m³ m³ = 1.086374
m = 1.028 = 102.8%
x = 2.8%

28th May

(a) Sketch the graph of $y = \sin 2x$ between $x = 0$ and $x = 360°$.
(b) Solve the equation $\cos x = -0.44$ giving all possible answers between $x = 0$ and $x = 360°$

(a)



(b) $\cos x = -0.44$
$x = \cos^{-1}(-0.44) =$ **116.1°**
2nd answer: 360 – 116.1 = **243.9°**

29th May

The table shows the times 106 shoppers spent in a supermarket.

Time (t minutes)	Frequency
$0 < t \leq 10$	20
$10 < t \leq 20$	17
$20 < t \leq 30$	12
$30 < t \leq 40$	32
$40 < t \leq 50$	25

Find the group the median lies in and estimate the mean time.

Median = 53rd and 54th piece of data.
Both lie in the $30 < t \leq 40$ group

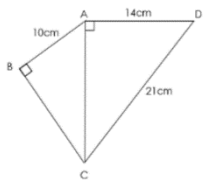
$$\text{Mean} = \frac{(20\times5)+(17\times15)+(12\times25)+(32\times35)+(25\times45)}{106} = \frac{2900}{106} = \mathbf{27.36}$$

30th May

In the formula $v^2 = u^2 + 2as$
$v = 9.4$, $a = -1.12$ and $s = 23.4$
Find u

$9.4^2 = u^2 + 2(-1.12)(23.4)$
$88.36 = u^2 - 52.416$ $\quad u^2 = 140.776$ $\quad$ **$u = 11.86$**

31st May



Find the size of angle BAC

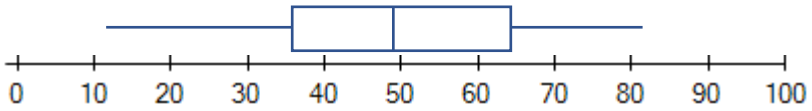
$AC^2 + 14^2 = 21^2$ $\quad AC^2 + 196 = 441$ $\quad AC = \sqrt{245}$

$\cos BAC = \frac{10}{\sqrt{245}}$ BAC $= \cos^{-1}\left(\frac{10}{\sqrt{245}}\right) =$ **50.29°**

1st June

A year 11 class has done a mathematics test. The following information has been found: Highest mark was 81. Half the class got less than 49 marks. 25% of the class got more than 69 marks. 75% of the class got more than 35 marks. The range of marks was 70.
Draw a box and whisker diagram to illustrate the information.

Lowest = 81 – 70 = 11 $\quad$ Highest = 81
$Q_1 = 35$ $\quad Q_2 = 49$ $\quad Q_3 = 69$



2nd June

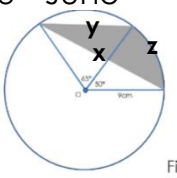
$f(x) = 2x^4 - 3x^2 + 5x - 2$
(a) Show there is a solution to f(x) = 0 between -1 and -2.
(b) Show that f(x) = 0 can be rearranged to give $x = \sqrt[3]{\frac{3x^2 - 5x + 2}{2x}}$
(c) Using the iterative formula $x_{n+1} = \sqrt[3]{\frac{3x_n^2 - 5x_n + 2}{2x_n}}$
And $x_0 = -1.5$, find x_4 to 4 d.p.

(a) f(-1) = -8 $\quad$ f(-2) = 8 $\quad$ Change of sign so solution between -1 & -2

(b) $2x^4 = 3x^2 - 5x + 2$ $\quad x^3 = \frac{3x^2-5x+2}{2x}$ $\quad x = \sqrt[3]{\frac{3x^2-5x+2}{2x}}$

(c) $x_0 = -1.5$ $\quad x_1 = 1.75621$ $\quad x_2 = 1.78671$ $\quad x_3 = 1.79046$ $\quad$ **$x_4 = 1.7909$**

3rd June



Find the area and perimeter of the shaded region.

Area of large sector $= \frac{115}{360} \times \pi \times 9^2 = 81.29$
Triangle $= \frac{1}{2} \times 9 \times 9 \times \sin 115 = 36.71$
Segment at top $= \left(\frac{65}{360} \times \pi \times 9^2\right) - \frac{1}{2} \times 9 \times 9 \times \sin 65 = 9.24$
Shaded Area = 81.29 – 36.71 – 9.24 = **35.34cm²**
$x^2 = 9^2 + 9^2 - 2(9)(9)\cos 115$ $\quad x = 15.18$cm
$y^2 = 9^2 + 9^2 - 2(9)(9)\cos 65$ $\quad y = 9.67$cm $\quad z = \frac{50}{360} \times \pi \times 18 = 7.85$
Perimeter = 15.18 + 9.67 + 7.85 = **32.7cm**