HIGHER QUESTIONS FOR PAPERS 2 AND 3

NUMBER

1. Toby invested £7500 for 2 years in a savings account.
He was paid 4% per annum compound interest.
How much money did Toby have in his savings account at the end of 2 years?

2. The length, L cm, of a line is measured as 13 cm correct to the nearest centimetre.
Complete the following statement to show the range of possible values of L.

..... $\leq L <$ .....

3. $D = \dfrac{x}{y}$ x = 99.7 correct to 1 decimal place. y = 67 correct to 2 significant figures.
Work out an upper bound for D.

4. Ian invested an amount of money at 3% per annum compound interest.
At the end of 2 years the value of the investment was £2652.25.
(*a*) Work out the amount of money Ian invested.
Noah has an amount of money to invest for five years.

Saver Account	Investment Account
4% per annum compound interest.	21% interest paid at the end of 5 years.

Noah wants to get the most interest possible.
(*b*) Which account is best? You must show how you got your answer.

5. There are 14 boys and 12 girls in a class.
Work out the total number of ways that 1 boy and 1 girl can be chosen from the class.

6. Jim rounds a number, x, to one decimal place.
The result is 7.2. Write down the error interval for x.

7. (*a*) Find the reciprocal of 2.5.

(*b*) Work out $\sqrt[3]{\dfrac{4.3 \times \tan 39°}{23.4 - 6.06}}$ Give your answer correct to 3 significant figures.

8. The surface gravity of a planet can be worked out using the formula

$$g = \frac{6.67 \times 10^{-11} m}{r^2}$$

Where m kilograms is the mass of the planet r metres is the radius of the planet
For the Earth and Jupiter here are the values of m and r.

Earth	Jupiter
$m = 5.98 \times 10^{24}$ $r = 6.378 \times 10^{6}$	$m = 1.90 \times 10^{27}$ $r = 7.149 \times 10^{7}$

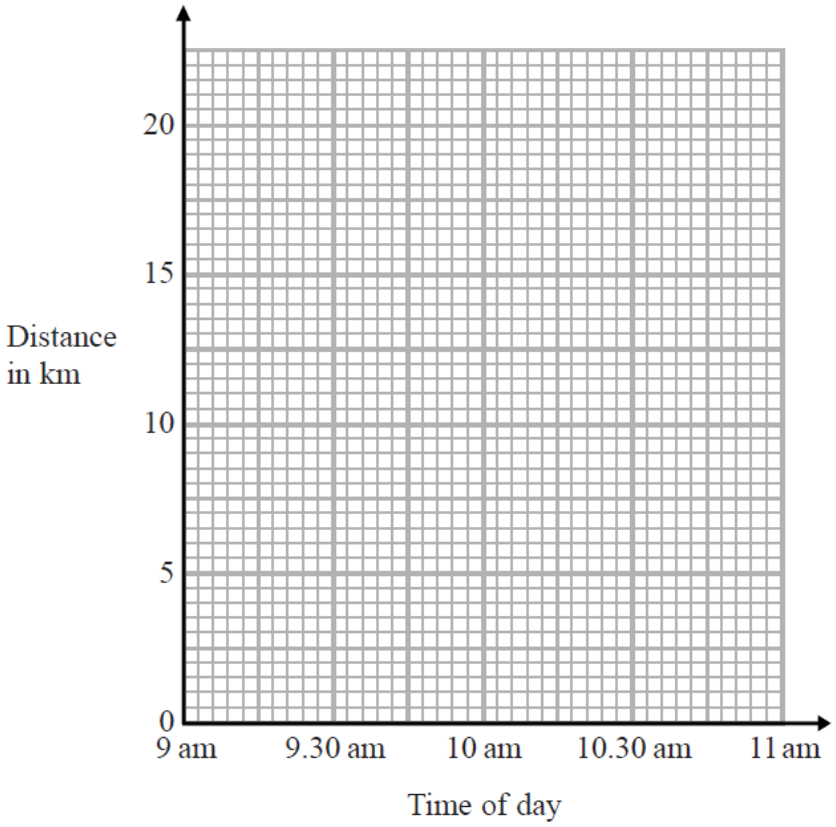
Work out the ratio of the surface gravity of Earth to the surface gravity of Jupiter.
Write your answer in the form 1 : n.

9. Prove algebraically that the recurring decimal $0.3\dot{1}\dot{8}$ can be written as $\frac{7}{22}$

ALGEBRA

1. Make t the subject of the formula $w = 3t + 11$

2. At 9 am, Bradley began a journey on his bicycle.
From 9 am to 9.36 am, he cycled at an average speed of 15 km/h.
From 9.36 am to 10.45 am, he cycled a further 8 km.
(*a*) Draw a travel graph to show Bradley's journey.



From 10.45 am to 11 am, Bradley cycled at an average speed of 18 km/h.
(*b*) Work out the distance Bradley cycled from 10.45 am to 11 am.

3. Becky has some marbles.
Chris has two times as many marbles as Becky. Dan has seven more marbles than Chris.
They have a total of 57 marbles.
Dan says,
"If I give some marbles to Becky, each of us will have the same number of marbles."
Is Dan correct?
You must show how you get your answer

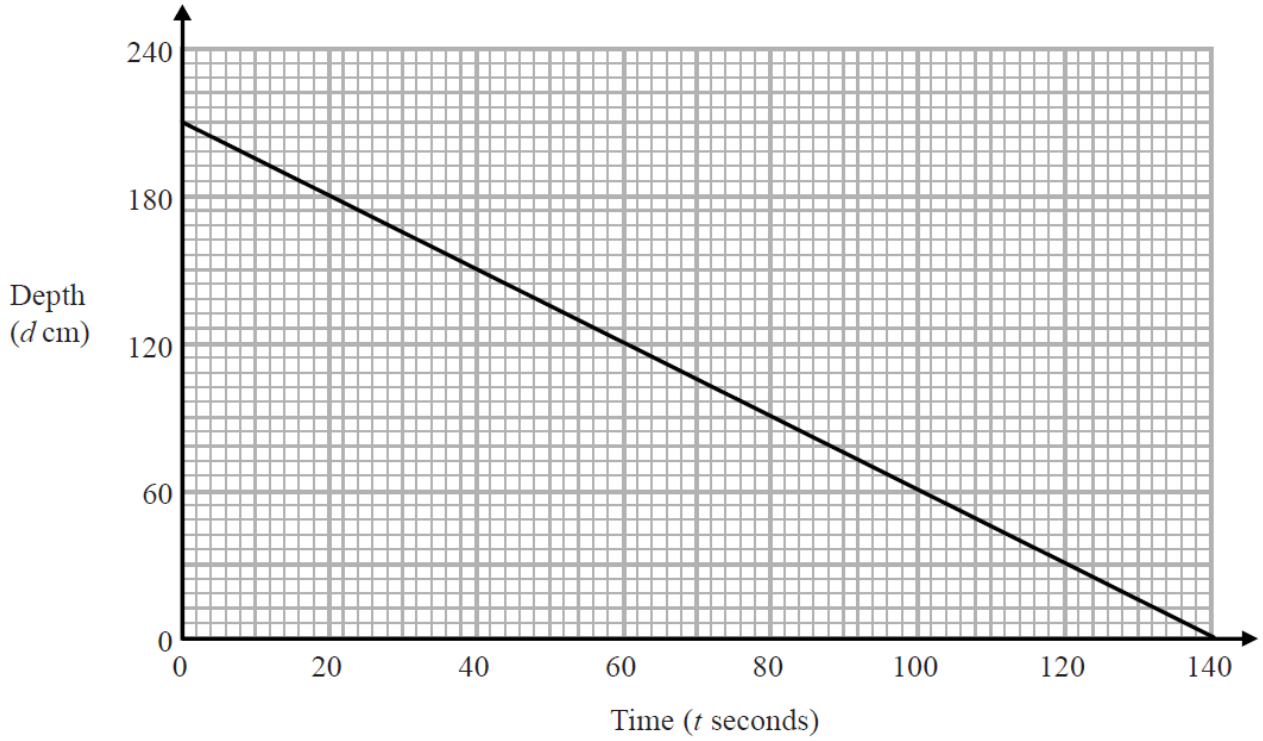
4. Factorise $x^2 + 3x - 4$

5. Here are the first 5 terms of a quadratic sequence.

1 3 7 13 21

Find an expression, in terms of n, for the nth term of this quadratic sequence.

6. The graph shows the depth, d cm, of water in a tank after t seconds.

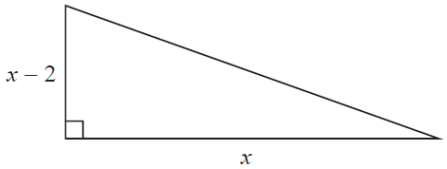


(a) Find the gradient of this graph.
(b) Explain what this gradient represents.

7. A pendulum of length L cm has time period T seconds.
T is directly proportional to the square root of L.
The length of the pendulum is increased by 40%.
Work out the percentage increase in the time period.

8. $f(x) = 3x^2 - 2x - 8$
Express $f(x + 2)$ in the form $ax^2 + bx$

9. Here is a right-angled triangle.

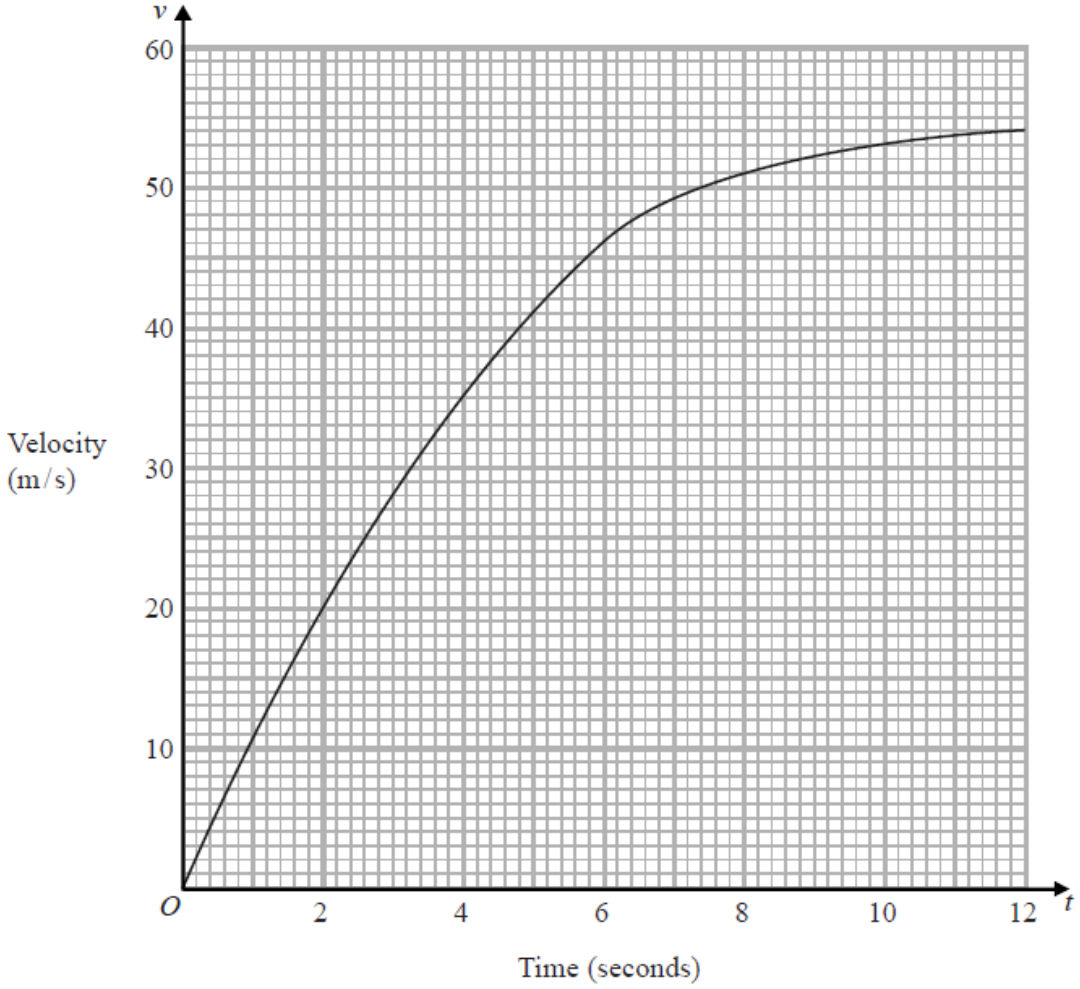


All measurements are in centimetres. The area of the triangle is 2.5 cm^2.
Find the perimeter of the triangle.
Give your answer correct to 3 significant figures.
You must show all of your working.

10. Solve the simultaneous equations
$2x - 4y = 19$
$3x + 5y = 1$

11. Write $x^2 + 2x - 8$ in the form $(x + m)^2 + n$ where m and n are integers.

12. The graph shows information about the velocity, v m/s, of a parachutist t seconds after leaving a plane.



(*a*) Work out an estimate for the acceleration of the parachutist at $t = 6$

(*b*) Work out an estimate for the distance fallen by the parachutist in the first 12 seconds after leaving the plane. Use 3 strips of equal width.

13. The number of bees in a beehive at the start of year n is P_n.
The number of bees in the beehive at the start of the following year is given by

$$P_{n+1} = 1.05(P_n - 250)$$

At the start of 2015 there were 9500 bees in the beehive.
How many bees will there be in the beehive at the start of 2018?

14. (*a*) Show that the equation $3x^2 - x^3 + 3 = 0$ can be rearranged to give $x = 3 + \frac{3}{x^2}$

(*b*) Using $x_{n+1} = 3 + \frac{3}{x_n^2}$ with $x_0 = 3.2$, find the values of x_1, x_2 and x_3

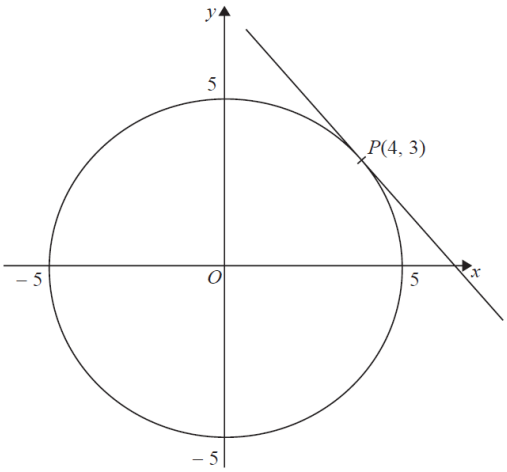
(*c*) Explain what the values of x_1, x_2 and x_3 represent.

15. Here are the first five terms of an arithmetic sequence.

7 13 19 25 31

Prove that the difference between the squares of any two terms of the sequence is always a multiple of 24.

16. Here is a circle, centre O, and the tangent to the circle at the point $P(4, 3)$ on the circle.

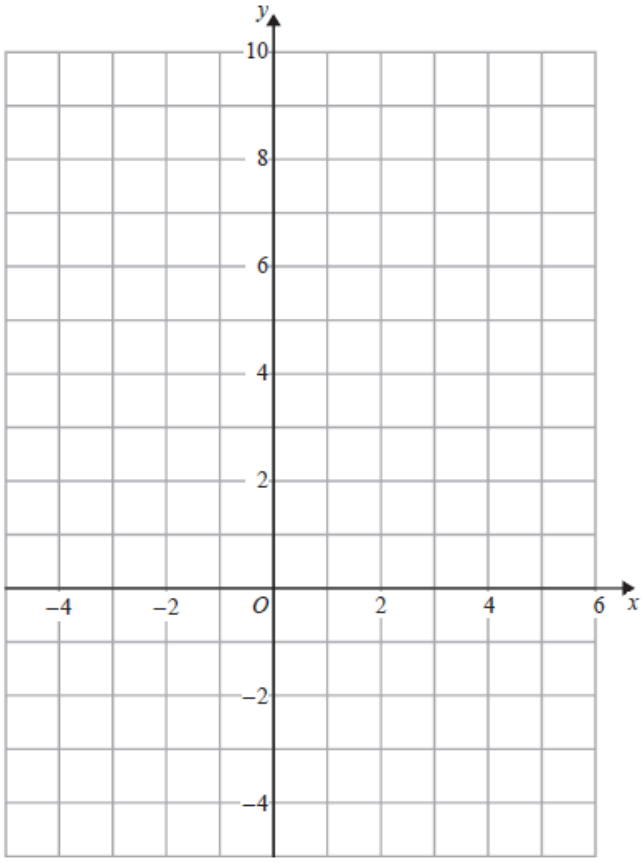


Find an equation of the tangent at the point P.

17. On the grid, shade the region that satisfies all these inequalities.

$x + y < 4$ $y > x - 1$ $y < 3x$

Label the region **R**.



18. Write $4-\left[(x+3)\div\dfrac{x^2+5x+6}{x-2}\right]$ as a single fraction in its simplest form.

You must show your working.

19. A virus on a computer is causing errors.
An antivirus program is run to remove these errors.

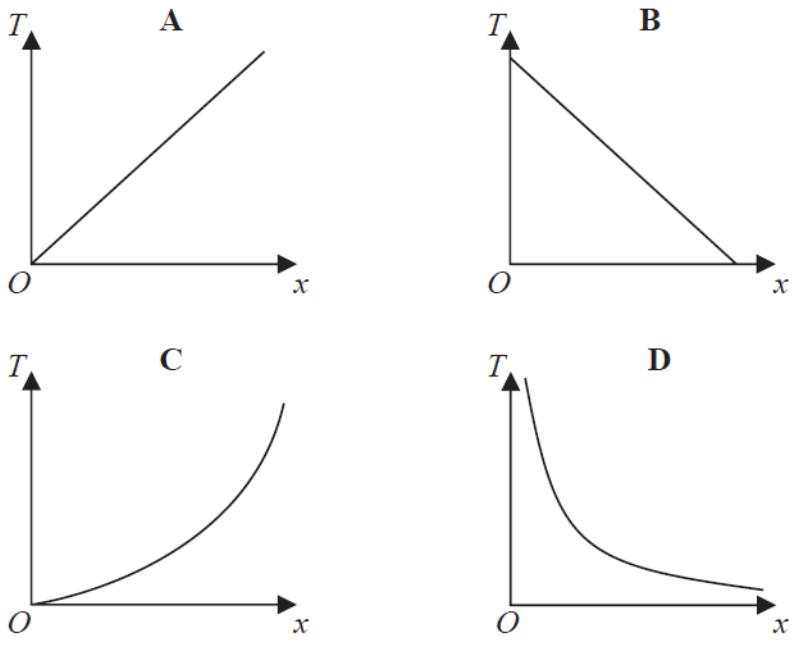
An estimate for the number of errors at the end of t hours is $10^6 \times 2^{-t}$
(*a*) Work out an estimate for the number of errors on the computer at the end of 8 hours.
(*b*) Explain whether the number of errors on this computer ever reaches zero.

20. The graph of $y = f(x)$ is transformed to give the graph of $y = -f(x + 3)$
The point A on the graph of $y = f(x)$ is mapped to the point P on the graph of $y = -f(x + 3)$
The coordinates of point A are (9, 1). Find the coordinates of point P.

21. (*a*) Expand and simplify $3(y - 2) + 5(2y + 1)$
(*b*) Simplify $5u^2w^4 \times 7uw^3$

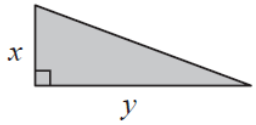
22. Make t the subject of the formula $y = \frac{t}{3} - 2a$

23. At a depth of x metres, the temperature of the water in an ocean is T °C.
At depths below 900 metres, T is inversely proportional to x.
T is given by $T = \frac{4500}{x}$
(*a*) Work out the difference in the temperature of the water at a depth of 1200 metres and the temperature of the water at a depth of 2500 metres.
Here are four graphs.

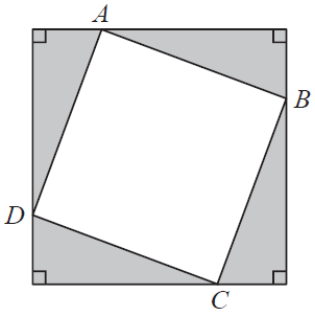


One of the graphs could show that T is inversely proportional to x.
(*b*) Write down the letter of this graph.

24. Here is a right-angled triangle.



Four of these triangles are joined to enclose the square *ABCD* as shown below.

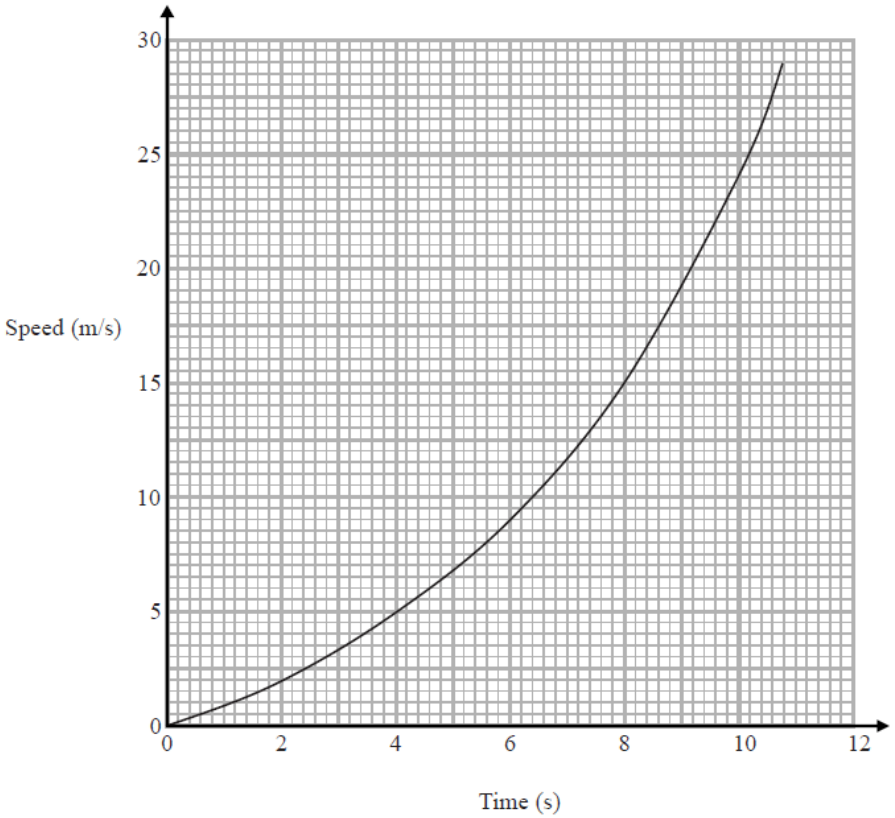


Show that the area of the square *ABCD* is $x^2 + y^2$

25. Show that $\frac{a}{b+1} - \frac{a}{(b+1)^2}$ can be written as $\frac{ab}{(b+1)^2}$

26. The product of two consecutive positive integers is added to the larger of the two integers. Prove that the result is always a square number.

27. Here is a speed-time graph for a car.



(*a*) Work out an estimate for the distance the car travelled in the first 10 seconds.
Use 5 strips of equal width.

(*b*) Is your answer to (*a*) an underestimate or an overestimate of the actual distance?
Give a reason for your answer.

28. (*a*) Write $2x^2 + 16x + 35$ in the form $a(x + b)^2 + c$ where a, b, and c are integers.

(*b*) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 2x^2 + 16x + 35$

RATIO AND PROPORTION

1. Three companies sell the same type of furniture.
The price of the furniture from Pooles of London is £1480
The price of the furniture from Jardins of Paris is €1980
The price of the furniture from Outways of New York is $2250
The exchange rates are

£1 = €1.34
£1 = $1.52

Which company sells this furniture at the lowest price?
You must show how you get your answer.

2. The densities of two different liquids A and B are in the ratio 19 : 22
The mass of 1 cm^3 of liquid B is 1.1 g.
5 cm^3 of liquid A is mixed with 15 cm^3 of liquid B to make 20 cm^3 of liquid C.
Work out the density of liquid C.

3. On a farm
the number of cows and the number of sheep are in the ratio 6 : 5
the number of sheep and the number of pigs are in the ratio 2 : 1
The total number of cows, sheep and pigs on the farm is 189.
How many sheep are there on the farm?

4. The ratio of the number of boys to the number of girls in a school is 4 : 5
There are 95 girls in the school.

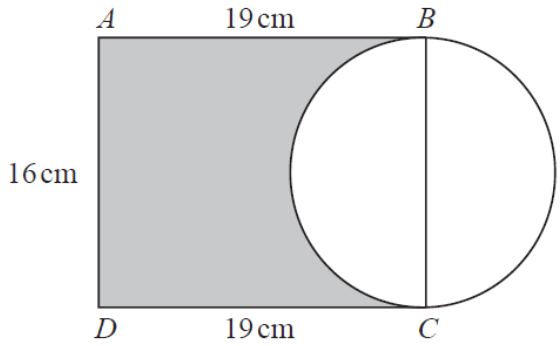
Work out the total number of students in the school.

5. Ibrar bought a house for £145 000.
The value of the house depreciated by 4% in the first year.
The value of the house depreciated by 2.5% in the second year.
Ibrar says,
"4 + 2.5 = 6.5 so in two years the value of my house depreciated by 6.5%"
(*a*) Is Ibrar right?
You must give a reason for your answer.
The value of Ibrar's house increases by x% in the third year.
At the end of the third year the value of Ibrar's house is £140 000.
(*b*) Work out the value of x. Give your answer correct to 3 significant figures.

6. Zahra mixes 150 g of metal A and 150g of metal B to make 300 g of an alloy.
Metal A has a density of 19.3 g/cm^3.
Metal B has a density of 8.9 g/cm^3.
Work out the density of the alloy.

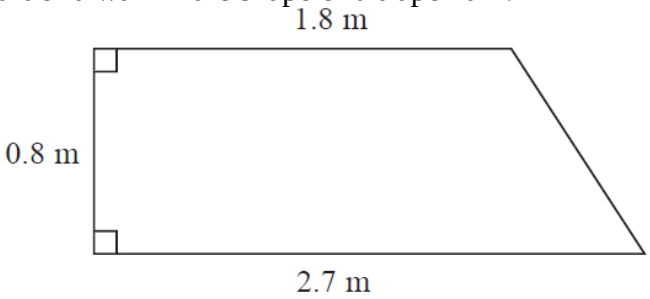
SHAPE

1. Here is a diagram showing a rectangle, *ABCD*, and a circle.



BC is a diameter of the circle.
Calculate the percentage of the area of the rectangle that is shaded.
Give your answer correct to 1 decimal place.

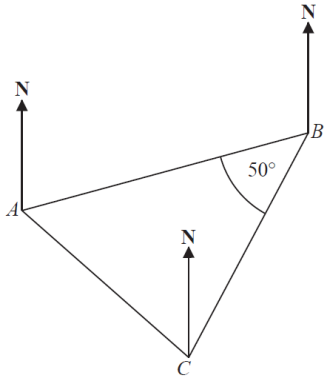
2. The diagram shows part of a wall in the shape of a trapezium.



Karen is going to cover this part of the wall with tiles.
Each rectangular tile is 15 cm by 7.5 cm.
Tiles are sold in packs.
There are 9 tiles in each pack.
Karen divides the area of the wall by the area of a tile to work out an estimate for the number of tiles she needs to buy.
(*a*) Use Karen's method to work out an estimate for the number of packs of tiles she needs to buy.
Karen is advised to buy 10% more tiles than she estimated.
Buying 10% more tiles will affect the number of the tiles Karen needs to buy.
She assumes she will need to buy 10% more packs of tiles.
(*b*) Is Karen's assumption correct?
You must show your working.

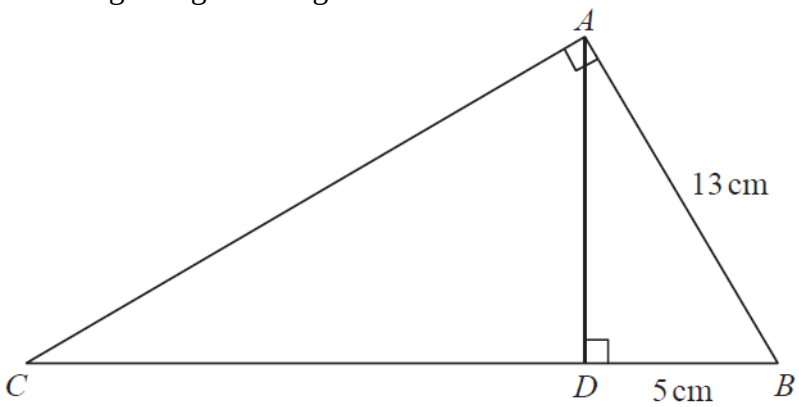
3. Mark has made a clay model.
He will now make a clay statue that is mathematically similar to the clay model.
The model has a base area of 6 cm^2.
The statue will have a base area of 253.5 cm^2.
Mark used 2 kg of clay to make the model.
Clay is sold in 10 kg bags.
Mark has to buy all the clay he needs to make the statue.
How many bags of clay will Mark need to buy?

4. The diagram shows the positions of three points, *A*, *B* and *C*, on a map.



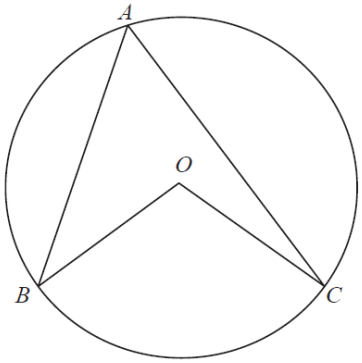
The bearing of *B* from *A* is 070° Angle *ABC* is 50° *AB* = *CB*
Work out the bearing of *C* from *A*.

5. *ABC* and *ABD* are two right-angled triangles.



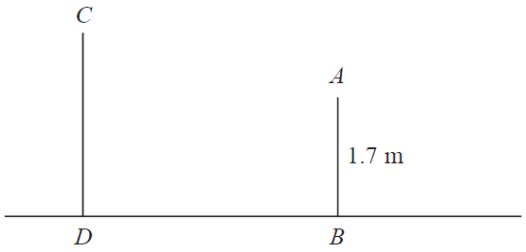
Angle *BAC* = angle *ADB* = 90° *AB* = 13 cm *DB* = 5 cm
Work out the length of *CB*.

6. *A*, *B* and *C* are points on the circumference of a circle centre *O*.



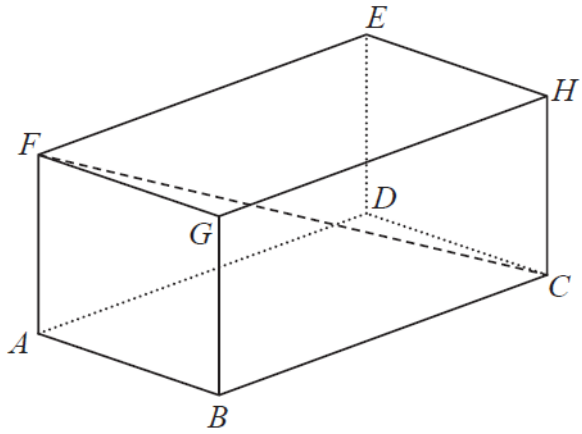
Prove that angle *BOC* is twice the size of angle *BAC*.

7. The diagram shows two vertical posts, *AB* and *CD*, on horizontal ground.



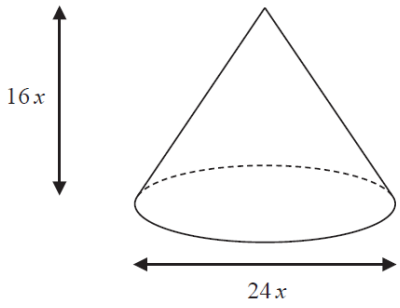
AB = 1.7 m *CD* : *AB* = 1.5 : 1 The angle of elevation of *C* from *A* is 52°
Calculate the length of *BD*. Give your answer correct to 3 significant figures.

8. The diagram shows a cuboid *ABCDEFGH*.



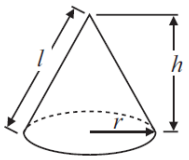
AB = 7 cm, *AF* = 5 cm and *FC* = 15 cm.
Calculate the volume of the cuboid. Give your answer correct to 3 significant figures.

9. The diagram shows a solid cone.



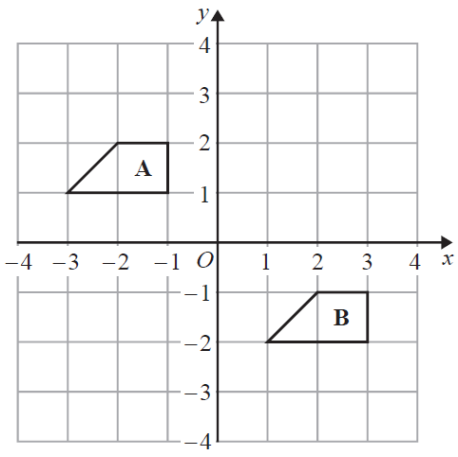
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$



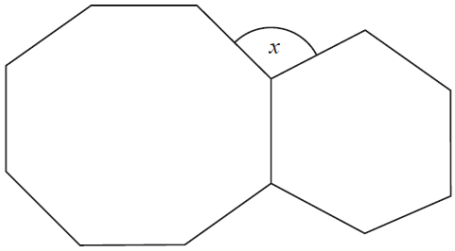
The diameter of the base of the cone is 24*x* cm. The height of the cone is 16*x* cm.
The curved surface area of the cone is 2160π cm^2.
The volume of the cone is $V\pi$ cm^3, where *V* is an integer.
Find the value of *V*.

10.



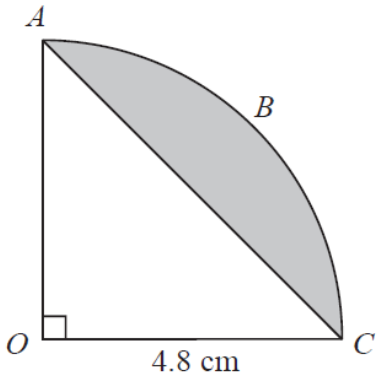
Describe the single transformation that maps shape **A** onto shape **B**.

11.



The diagram shows a regular octagon and a regular hexagon.
Find the size of the angle marked *x*. You must show all your working.

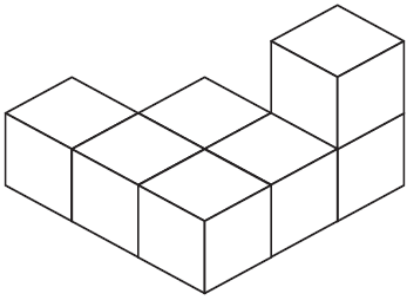
12.



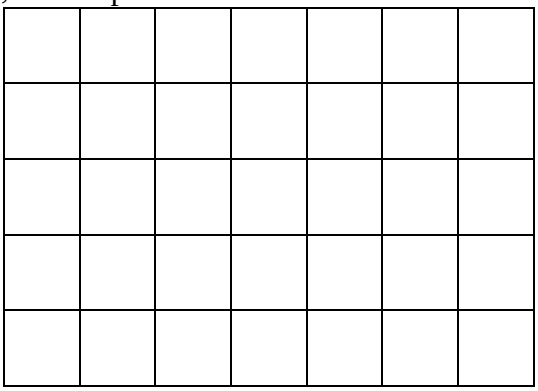
The arc *ABC* is a quarter of a circle with centre *O* and radius 4.8 cm.
AC is a chord of the circle.

Work out the area of the shaded segment.
Give your answer correct to 3 significant figures.

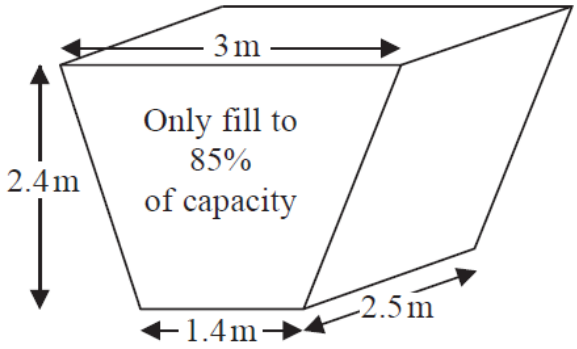
13. The diagram represents a solid made from seven centimetre cubes.



On the centimetre grid below, draw a plan of the solid.



14. The diagram shows an oil tank in the shape of a prism.
The cross section of the prism is a trapezium.



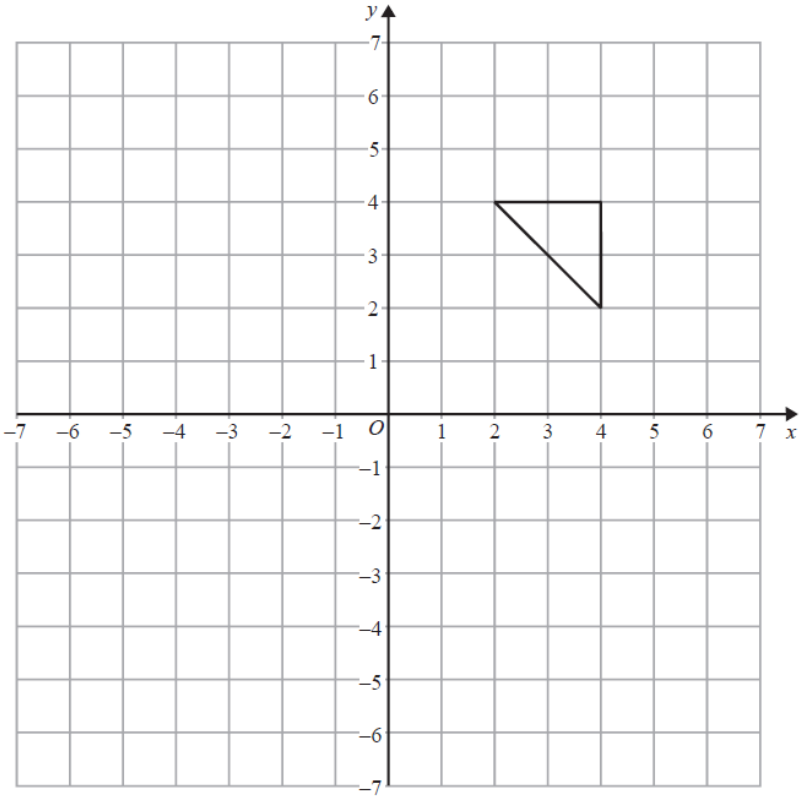
The tank is empty. Oil flows into the tank.
After one minute there are 300 litres of oil in the tank.
Assume that oil continues to flow into the tank at this rate.

(*a*) Work out how many **more** minutes it takes for the tank to be 85% full of oil.
($1\ m^3 = 1000$ litres)

The assumption about the rate of flow of the oil could be wrong.

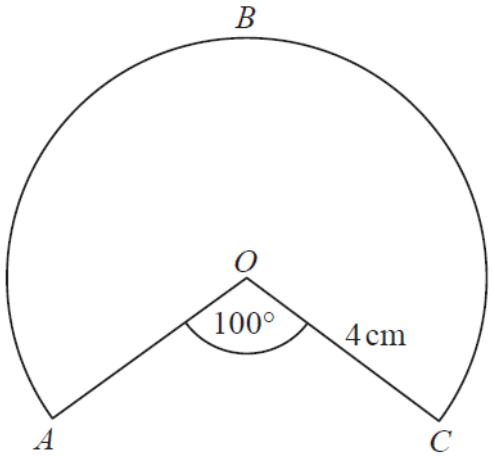
(*b*) Explain how this could affect your answer to part (*a*).

15.

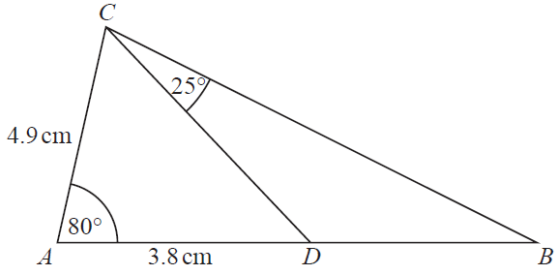


On the grid, enlarge the triangle by scale factor $-1\frac{1}{2}$, centre (0, 2).

16. The diagram shows a sector of a circle of radius 4 cm.



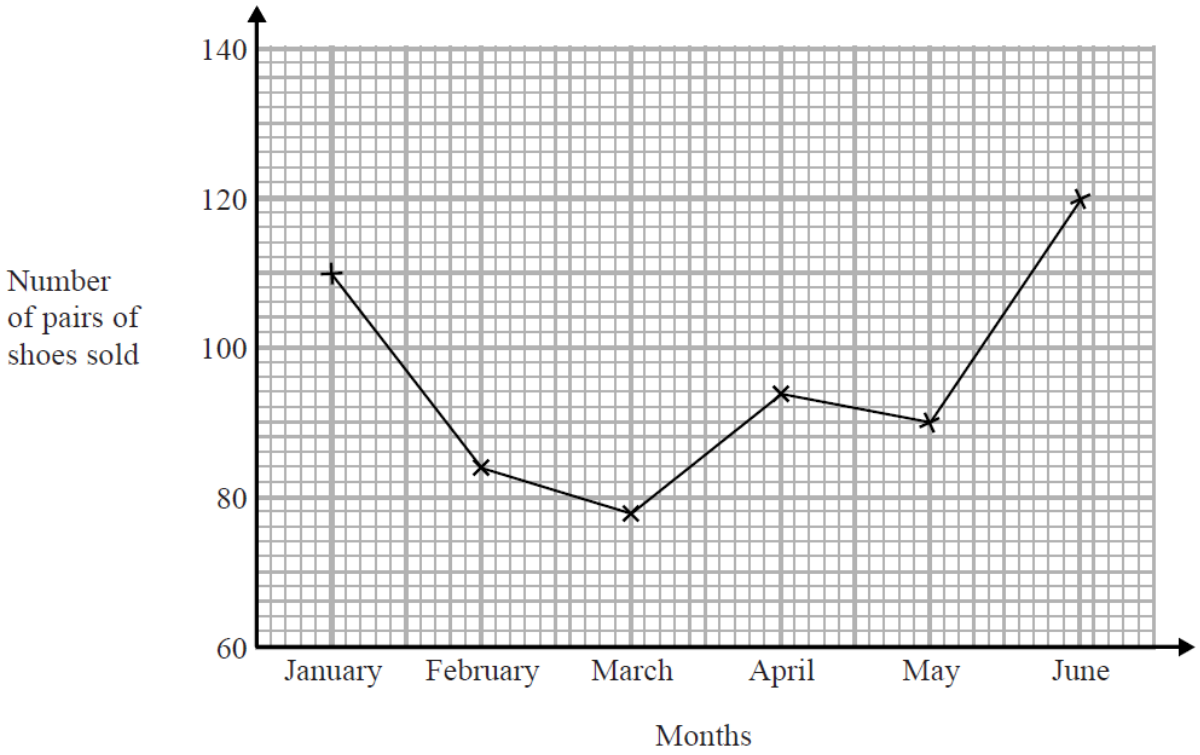
17. Work out the length of the arc *ABC*. Give your answer correct to 3 significant figures.



ABC is a triangle. *D* is a point on *AB*.
Work out the area of triangle *BCD*. Give your answer correct to 3 significant figures.

HANDLING DATA AND PROBABILITY

1. The time-series graph gives some information about the number of pairs of shoes sold in a shoe shop in the first six months of 2014.



The sales target for the first six months of 2014 was to sell a mean of 96 pairs of shoes per month.
Did the shoe shop meet this sales target? You must show how you get your answer.

2. Finlay plays two tennis matches.
The probability that he will win a match is 0.7. Win and lose are the only options.
The probability of winning a match is constant.
(*a*) Work out the probability that Finlay wins both matches.
(*b*) Work out the probability that Finlay loses at least one match.

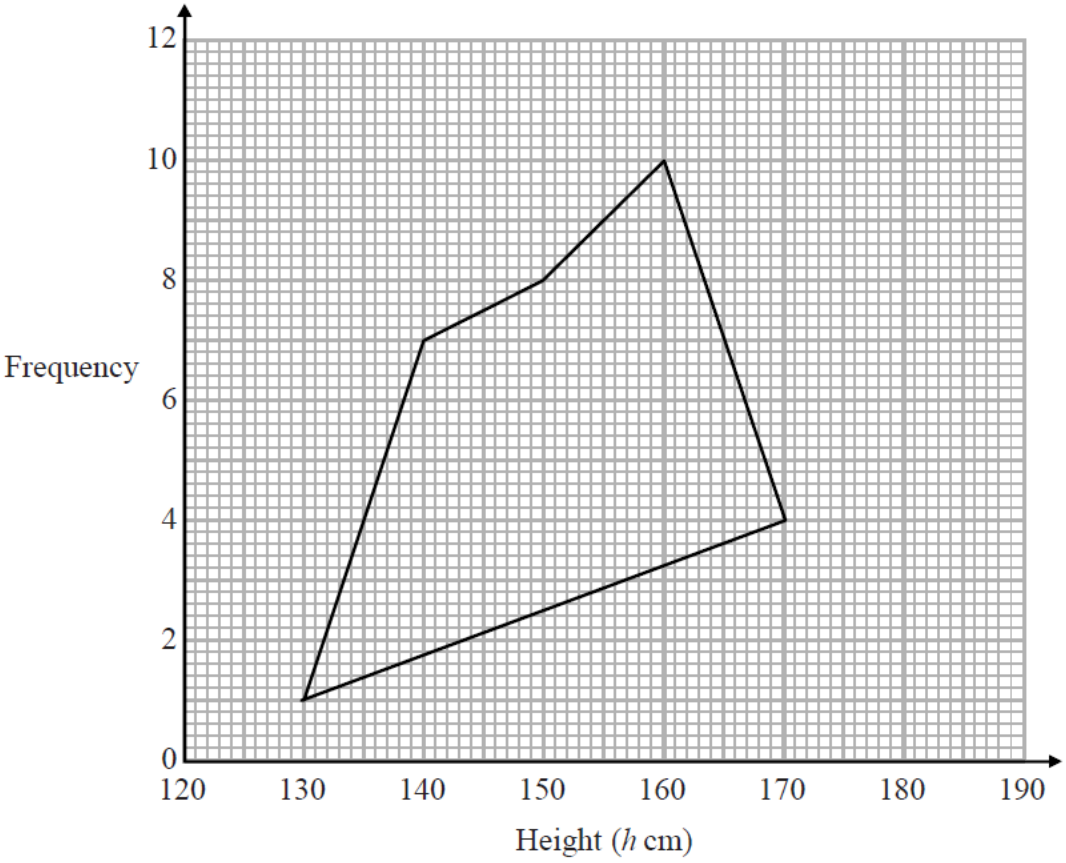
3. $\mathscr{E}$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {multiples of 2}
$A \cap B$ = {2, 6}
$A \cup B$ = {1, 2, 3, 4, 6, 8, 9, 10}
Draw a Venn diagram for this information.

4. The grouped frequency table gives information about the heights of 30 students.

Height (h cm)	**Frequency**
$130 < h \leq 140$	1
$140 < h \leq 150$	7
$150 < h \leq 160$	8
$160 < h \leq 170$	10
$170 < h \leq 180$	4

(*a*) Write down the modal class interval.

This incorrect frequency polygon has been drawn for the information in the table.



(*b*) Write down two things wrong with this incorrect frequency polygon.

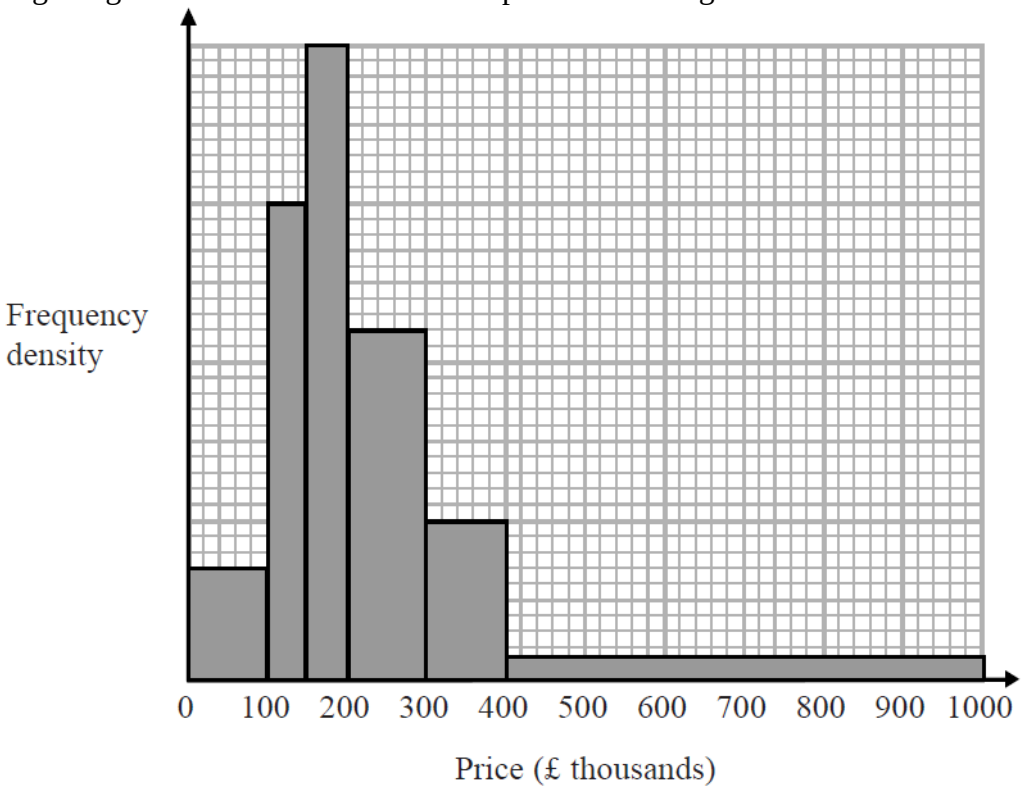
(*c*) Estimate the mean height of the students.

5. Jenny works in a shop that sells belts.
The table shows information about the waist sizes of 50 customers who bought belts from the shop in May.

Belt size	**Waist (*w* inches)**	**Frequency**
Small	$28 < w \leq 32$	24
Medium	$32 < w \leq 36$	12
Large	$36 < w \leq 40$	8
Extra Large	$40 < w \leq 44$	6

(*a*) Calculate an estimate for the mean waist size.
Belts are made in sizes Small, Medium, Large and Extra Large.
Jenny needs to order more belts in June.
The modal size of belts sold is Small.
Jenny is going to order $\frac{3}{4}$ of the belts in size Small.
The manager of the shop tells Jenny she should **not** order so many Small belts.
(*b*) Who is correct, Jenny or the manager?
You must give a reason for your answer.

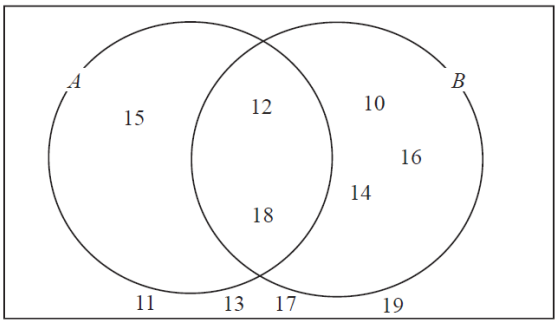
6. The histogram gives information about house prices in a village in 2015.



20 houses in the village have a price between £300 000 and £400 000.
Work out the number of houses in the village with a price under £200 000.

7. Thelma spins a biased coin twice.
The probability that it will come down heads both times is 0.09
Calculate the probability that it will come down tails both times.

8. Here is a Venn diagram.



(*a*) Write down the numbers that are in set

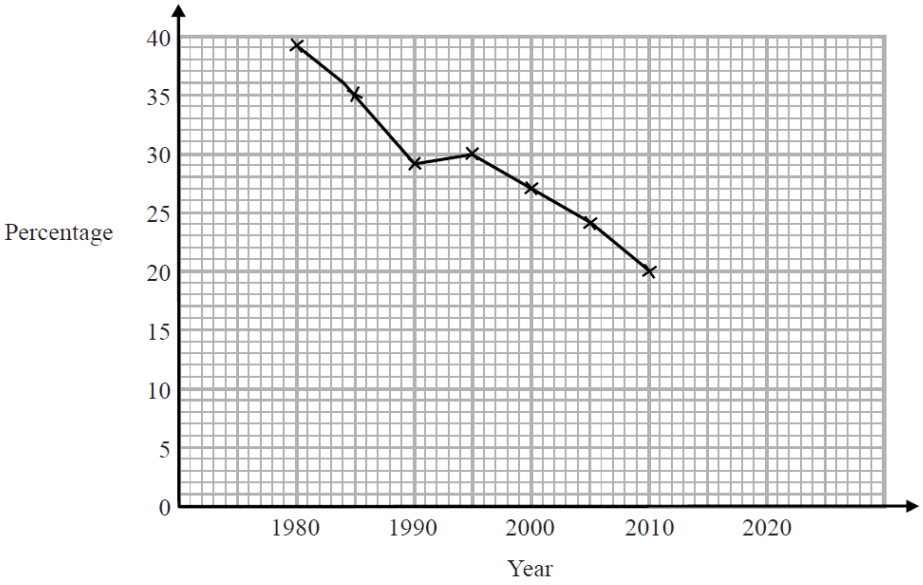
(i) $A \cup B$

(ii) $A \cap B$

One of the numbers in the diagram is chosen at random.

(*b*) Find the probability that the number is in set A'

9. The time series graph shows information about the percentages of the people in a village that used the village shop for the years between 1980 and 2010.



(*a*) Describe the trend in the percentage of the people in the village who used the shop for this period.

(*b*) (i) Use the graph to predict the percentage of the people in the village likely to use the shop in the year 2020.

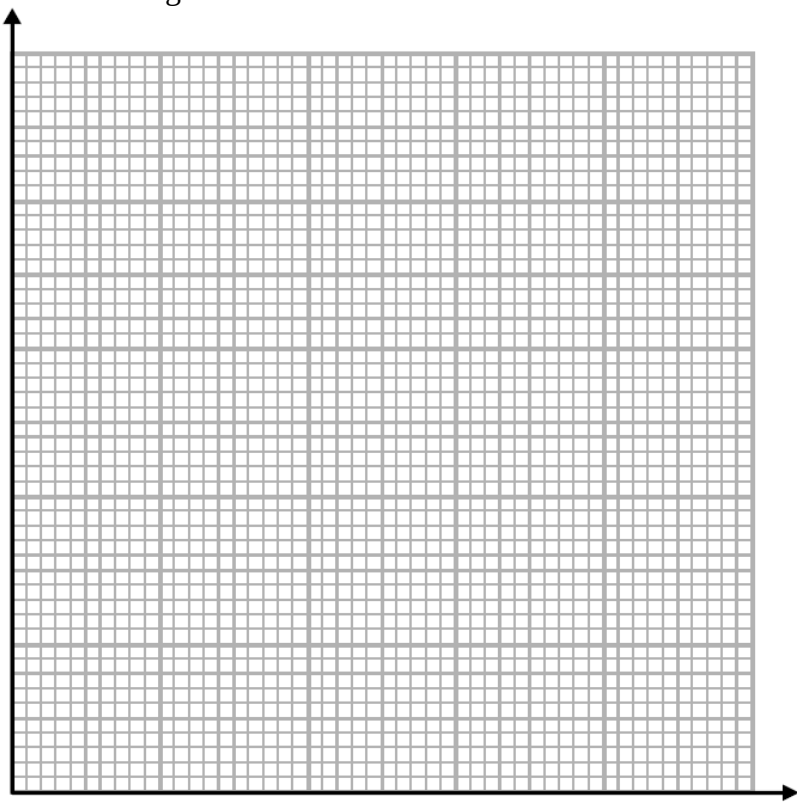
(ii) Is your prediction reliable?

Explain your answer.

10. The table gives information about the speeds, in km/h, of 81 cars.

Speed (s km/h)	**Frequency**
$90 < s \leq 100$	13
$100 < s \leq 105$	16
$105 < s \leq 110$	18
$110 < s \leq 120$	22
$120 < s \leq 140$	12

(*a*) On the grid, draw a histogram for the information in the table.



(*b*) Find an estimate for the median.

11. There are y black socks and 5 white socks in a drawer.
Joshua takes at random two socks from the drawer.

The probability that Joshua takes one white sock and one black sock is $\frac{6}{11}$

(*a*) Show that $3y^2 - 28y + 60 = 0$
(*b*) Find the probability that Joshua takes two black socks.